Virginia Tech Regional Math Contest 1993*

Problem 1. Prove that $\displaystyle\int_0^1 \int_{x^2}^1 e^{y^{3/2}}\,dy\,dx = \frac{2e-2}{3}$.

Problem 2. Prove that if $f : \mathbb{R} \to \mathbb{R}$ is continuous and $f(x) = \displaystyle\int_0^x f(t)\,dt$, then $f(x)$ is identically zero.

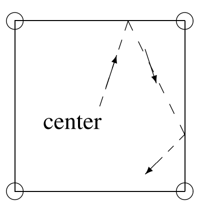
Problem 3. Let $f_1(x) = x$ and $f_{n+1}(x) = x^{f_n(x)}$, for $n = 1, 2 \ldots$. Prove that $f_n'(1) = 1$ and $f_n''(1) = 2$, for all $n \geq 2$.

Problem 4. Prove that a triangle in the plane whose vertices have integer coordinates cannot be equilateral.

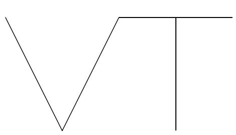
Problem 5. Find $\displaystyle\sum_{n=1}^{\infty} \frac{3^{-n}}{n}$.

Problem 6. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a surjective map with the property that if the points A, B and C are collinear, then so are $f(A)$, $f(B)$ and $f(C)$. Prove that f is bijective.

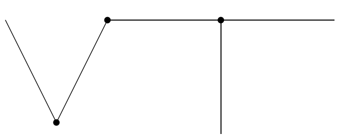
Problem 7. On a small square billiard table with sides of length 2 ft., a ball is played from the center and after rebounding off the sides several times, goes into a cup at one of the corners. Prove that the total distance travelled by the ball is **not** an integer number of feet.



Problem 8. A popular Virginia Tech logo looks something like



Suppose that wire-frame copies of this logo are constructed of 5 equal pieces of wire welded at three places as shown:



If bending is allowed, but no re-welding, show clearly how to cut the maximum possible number of ready-made copies of such a logo from the piece of welded wire mesh shown. Also, prove that no larger number is possible.

*Source: https://personal.math.vt.edu/plinnell/Vtregional/