

Virginia Tech Regional Math Contest 2003¹

Problem 1

An investor buys stock worth and holds it for n business days. Each day he has an equal chance of either gaining 20% or losing 10%. However, in the case he gains every day (i.e., n gains of 20%), he is deemed to have lost all his money, because he must have been involved with insider trading. Find a (simple) formula, with proof, of the amount of money he will have on average at the end of the n days.

Problem 2

Find $\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)} = \frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \dots$ for $|x| < 1$.

Problem 3

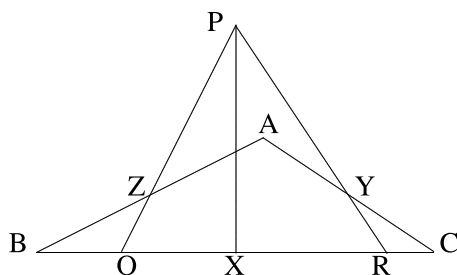
Determine all invertible 2 by 2 matrices A with complex numbers as entries satisfying $A = A^{-1} = A^{\top}$, where $A^{\top}$ denotes the transpose of A .

Problem 4

It is known that $2 \cos^3 \frac{\pi}{7} - \cos^2 \frac{\pi}{7} - \cos \frac{\pi}{7}$ is a rational number. Write this rational number in the form $\frac{p}{q}$, where p and q are integers with q positive.

Problem 5

In the diagram below, X is the midpoint of BC , Y is the midpoint of AC , and Z is the midpoint of AB . Also $\angle ABC + \angle PQC = \angle ACB + \angle PRB = 90^\circ$. Prove that $\angle PXC = 90^\circ$.



Problem 6

Let $f : [0, 1] \rightarrow [0, 1]$ be a continuous function such that $f(f(f(x))) = x$ for all $x \in [0, 1]$. Prove that $f(x) = x$ for all $x \in [0, 1]$. Here $[0, 1]$ denotes the closed interval of all real numbers between 0 and 1, including 0 and 1.

Problem 7

Let T be a solid tetrahedron whose edges all have length 1. Determine the volume of the region consisting of points which are at distance at most 1 from some point in T (your answer should involve $\sqrt{2}$, $\sqrt{3}$, π).

¹VTRMC problems source: <https://personal.math.vt.edu/plinnell/Vtregional/>